AMERICAN UNIVERSITY OF BEIRUT
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT
EECE 340
Homework VI – State Variables Analysis

Problem 1

Let h(t) be the impulse response of a LTI system and its Laplace transform is given by:

$$H(s) = \frac{10(-s+1)}{(s+10)(s+1)}$$

a. Find the differential equation describing the system
b. Represent this system in a state-space form by writing the state equations and the output equation .

Problem 2

a. Draw the block diagram for a circuit that implements the following transfer function. You may use amplifiers, adders, and integrators.

$$\frac{Y(s)}{X(s)} = \frac{4s^2 + 6s + 12}{12s^3 + 2s^2 + 6s - 5}$$

b. Determine the state equation for this system.
c. Determine the output equation for this system.
d. Is the state representation for an LTI system unique? Explain.

Problem 3

The dynamic equations of a linear time-invariant system whose output is y(t) and input r(t) are given by:

$$\begin{bmatrix} X_1'(t) \\ X_2'(t) \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} r(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \end{bmatrix}$$

Determine the Differential equation representing this system.

Problem 4

A LTI system with input r(t) and output y(t) is represented by the DE equation

$$\frac{d^3y(t)}{dt^3} - 2\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} - 10y(t) = -2\frac{d^2r(t)}{dt^2} + 5r(t)$$

Write the state and the output equations of the system

Problem 5

The open-loop transfer function of a unity feedback control system is given by:

$$G(s) = \frac{3s + 10}{s^3 + 2s^2 + s}$$

1. Determine the Differential equation representing this system.
2. Represent this system in a state variable form by determining the state equations and the output equation in matrix format.

Problem 6

A LTI system with input y(t) and output y(t) is represented by the DE

$$\frac{d^2y(t)}{dt^2} - 3\frac{dy(t)}{dt} + 6y(t) = \frac{d^2x(t)}{dt^2} - 12x(t)$$

a. Determine the system transfer function
b. Represent the system is a state space format.

Problem 7

Consider the following state-space representation:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t),$$

$$y(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}.$$

Obtain the system transfer function by using t**wo different methods**.

Problem 8

A linear time-invariant system, whose output is y(t) and input r(t), is represented by the following transfer function

$$\frac{Y(s)}{R(s)} = \frac{10s+6}{2s^2+8s+56}$$

Write the state and output equations of this system

Problem 9

Consider an LTI system described by the following input-output given below with r(t) is the input signal and y(t) is the output signal

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} - 6y(t) = \frac{d^2r(t)}{dt^2} + 2r(t)$$

Find the state space representation in this system